First Palis-Balzan Symposium on Dynamical Systems IMPA, from June 25th until 29th, 2012

This symposium was part of the Project Palis-Balzan - Dynamical Systems, Chaotic Behaviour-Uncertainty, sponsored by the Balzan Foundation, since the award conferred by the Balzan Foundation in 2010 to Jacob Palis and by IMPA, supported by CAPES, CNPq and FAPERJ.
The project involves scientists from different regions of the world and lasts for five years. Its main goal is to advance the global conjecture on the finiteness of the number of attractors for typical dynamical systems. Some other important topics are: linear cocycles and Lyapunov exponents.

The coordinators of this project are Jacob Palis, and Jean-Christophe Yoccoz, 1994 - Fields Medal Winner. As participants there are about 20 other renowned mathematicians, from Brazil (Artur Avila, Carlos Gustavo Moreira, Enrique Pujals, Lorenzo Diaz, Marcelo Viana, Jose Maria Pacífico, Welington de Melo), from France (Carlos Matheus, Sylvain Crovisier, Christian Bonatti and Pierre Berger), from U.S. (Marco Martens, Michael Lyubich), from England (Sebastian van Strien), from China (Lan Wen) and from Uruguay (Martin Sambarino and Alvaro Rovella), and about 10 young Ph.Ds.
This symposium aims to promote research at the highest level in the dynamical systems area, especially in the topics cited above, with the effective participation of excellent national and foreign researchers. It also aims to put the doctoral students and young researchers in contact with the best that is produced worldwide in the issues mentioned above and related ones, disseminating recent results and providing an international scientific exchange at the highest level. It stimulated the development of the Brazilian group in the area, which is increasingly asserting itself in the global context as one of the leading players.

For its realization, the International Conference was financially supported by:

- Balzan Foundation
- Conselho Nacional de Desenvolvimento Científico – CNPq
- Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro – FAPERJ
- Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – CAPES
- Instituto Nacional de Matemática Pura e Aplicada – IMPA
- Instituto Nacional de Ciência e Tecnologia de Matemática - INCTMAT

The Organizing Committee was composed by:

Artur Avila (IMPA / CNRS)
Sylvain Crovisier (CNRS)
Michael Lyubich (SUNY)
Welington Celso de Melo (IMPA)
Carlos Gustavo T. de A. Moreira (IMPA)
Jacob Palis (IMPA) - **Main Coordinator**
Enrique Pujals (IMPA)
Marcelo Viana (IMPA)
Jean Cristophe Yoccoz (Collége de France) - **Co-coordinator**

This conference was attended by about 107 researchers, including 26 foreign and 81 Brazilian (including a large number of masters and doctoral students from Rio de Janeiro who were not included in the participants list). The list of lectures and a partial list of participants are below.

Scientific Program:

Artur Ávila – IMPA and CNRS, France
On the metric properties of Feigenbaum-Julia sets
Pierre Berger – CNRS, France
Zoology in the Hénon family from twin baby Hénon-like attractors
Christian Bonatti – Université de Bourgogne, Dijon
Foliated hyperbolicity
Sylvain Crovisier – CNRS, France
Newhouse phenomenon and uniformity of extremal bundles
Lorenzo Diaz – PUC - Rio de Janeiro
Robust vanishing of all central Lyapunov exponents
Luiz Henrique de Figueiredo - IMPA, Rio de Janeiro
Images of Julia sets that you can trust
Nicolas Gourmelon - Université Bordeaux 1
C dichotomies between Newhouse phenomena and dominated splittings, at homoclinic points.
Pablo Guarino - Facultad de Ingenieria y Agrimesura
Rigidity of Critical Circle Map
Alejandro Kocsard - UFF,Niterói, RJ
Distributionally uniquely ergodic diffeomorphisms
Andrés Koropecki - UFF,Niterói, RJ
Prime ends rotation number and periodic points
Yuri Lima - Weizmann Institute
Stationary spaces of discrete groups: an Abramov formula
Jorge Eric López - IMPA
Stable projections of cartesian products of regular Cantor sets
Michael Lyubich – SUNY at Stony Brook , USA
On homoclinic tangencies in the complex Henon family
Marco Martens – SUNY at Stony Brook , USA
On the hyperbolicity of Lorenz renormalization
Carlos Gustavo Moreira – IMPA-RJ
On the continuity of fractal dimensions of horseshoes in dimenson 3
Sheldon Newhouse - Michigan State University, EUA
The Lorenz equations: A survey of rigorous results
Maria José Pacífico – Federal University, Rio de Janeiro
Fiber contracting maps versus Lorenz-like attractors
Vilton Pinheiro - UFBA, Bahia
Measures with historic behavior
Rafael Potrie - Centro de Matemática – Univ. de la República
Partial hyperbolicity and leaf conjugacy in nilmanifolds
Enrique Pujals – IMPA, Rio de Janeiro
Critical points for surfaces diffeomorphisms, abundance of periodic orbits and structural stability
Alvaro Rovella - Universidad de la República
Structural stability in dimension two
Martín Sambarino – Fac. de Ciencias del Uruguay
Some questions, problems and remarks regarding C^r dynamics
Carlos Matheus Santos – CNRS, France
Fractal geometry of non-uniformly hyperbolic horseshoes
Waliston Luiz Silva - Universidade Federal de São João Del-Rei
On the geometry of horseshoes
Sebastian Van Strien – Imperial College London
On stochastic stability of expanding circle maps with neutral fixed points
J. Regis Varão - USP – São Carlos
Center foliation: Absolute continuity, disintegration and rigidity
Marcelo Viana – IMPA, Rio de Janeiro
Time 1 maps of geodesic flows
Jiagang Yang - UFF Niterói, RJ
Diffeomorphisms with contracting Center

Mathematical Results presented at the Balzan-Palis Symposium:

Distributionally uniquely ergodic diffeomorphisms

By Alejandro Kocsard – Fluminense Federal University (UFF)

Abstract:
Invariant distributions appear as natural obstructions for the existence of smooth solutions for cohomological equations and they have been extensively used for estimating ergodic deviations, especially for certain parabolic and hyperbolic systems.

On the other hand, within elliptic systems, ergodic rigid translations on tori are prototypical examples of *distributionally uniquely ergodic* systems *DUE* for short, i.e. the Lebesgue measure is the only (up to multiplication by a constant) invariant distribution. Till very recently, they were the only known examples of DUE diffeomorphisms.

In this talk we shall discuss some general properties of DUE systems and present some new examples.

Structural stability in dimension two.

Álvaro Rovella – University of the Uruguayan Republic

Abstract:
We give a characterization of C1 stable endomorphisms in dimension two and show some examples.
Then, we study the topology of immediate basins, and give some ideas towards a classification of stable maps.

Prime ends rotation number and periodic points

Andrés Koropecki – Fluminense Federal University (UFF)

Abstract:
The boundary of a simply connected domain in a surface can be very complicated from the topological viewpoint (e.g. ``hairy", non-locally connected, etc). Carathéordory's theory of prime ends allows to compactify the simply connected domain, replacing its boundary by a circle, hence removing the topological complexity. When the domain is invariant by a surface homeomorphism, the dynamics extends to this circle of prime ends, and one can define its rotation number using the classic Poincaré theory. However, the dynamical consequences of this rotation number in the original space are not fully understood.
We consider this problem in the area preserving setting and other more general settings. Our main result establishes a complete classification linking the rationality of the rotation number with the existence of periodic points, and in addition provides topological information about the boundary of the domain. The main component states that, when the rotation number is irrational, then one of the following holds: (a) there is no periodic point in the boundary and the boundary is a contractible annular continuum, or (b) there is a unique fixed point, no other periodic points, and the boundary is a cellular continuum (contractible and non-separating). This includes the converse of a classic result of Cartwright and Littlewood, which considers the case of a rational rotation number.
Our main result has a number of consequences for generic area preserving surface diffeomorphisms. One of its applications is a novel proof of the following fact, which improves earlier work of Mather: for a C generic area preserving diffeomorphism, invariant (or periodic) complementary domains have no periodic points in their boundaries. This result was already known in the sphere and the torus (with a completely different proof), but not in surfaces of higher genus.

Remark: This is a joint work with Meysam Nassiri and Patrice Le Calvez.

Fractal geometry of non-uniformly hyperbolic horseshoes

Carlos Matheus Santos – CNRS and Université Paris 13

Abstract:
In his seminal works on Celestial Mechanics, H. Poincaré emphasized the relevance of homoclinic/heteroclinic bifurcations as a source of rich dynamical phenomena.

In a recent tour-de-force, J. Palis and J.-C. Yoccoz substantially advanced our understanding of heteroclinic bifurcations of surface diffeomorphisms by showing that certain objects called "non-uniformly hyperbolic horseshoes" are very common near heteroclinic bifurcations of “fat” horseshoes.

In this talk, we will discuss some results (obtained in collaboration with J. Palis and J.-C. Yoccoz) about the fractal geometry of these "non-uniformly hyperbolic horseshoes".

On the Continuity of Fractal Dimensions of Horseshoes on Dimenson 3

Carlos Gustavo Moreira - IMPA

Abstract:
We show that, for typical horseshoes in dimension 3, the Hausdorff dimensions of its stable and unstable Cantor sets, and of the whole horseshoe are continuous with respect to the dynamics.

Remark: This is a joint work with Waliston Silva.

Foliated hyperbolicity

Christian Bonatti - Université de Bourgogne - Dijon

Abstract:
Foliations in general do not admit invariant measure, leading to a difficulty for studying the statistical behavior of the leaves. A classical approach by L. Garnett considers harmonic measures which are the measures invariant under the Brownian notion in the leaves, and which are absolutely continuous in the leaves with a harmonic density.

With Xavier Gomez Mont and Matilde Martinez, we recently show that other absolutely continuous measure may describe the behavior of the geodesic in the leaves, when the leaves have negative curvature. For that we uses the hyperbolic properties of the geodesic flow in the leaves: this flow in neither hyperbolic, nor even partially hyperbolic, because the transverse behavior is not comparable with the tangent one. It is "foliated hyperbolic".

Our measure are the projections on the leaves of the foliated u-Gibbs states.
Using the properties of foliated hyperbolicity, Sebastien Alvarez exhibit a class of foliations which are uniquely ergodic for 3 meanings:

-- for the foliated brownian motion (harmonic measure)
-- for the geodesics (unique foliated u-Gibbs state)
-- for the large disks in the leaves (this comes from the unique ergodicity of the foliated Margulis flow).

In some specific cases, these three measures are distinct.

Square-tiled surfaces and generating the symmetric group

David Zmiaikou - Univ. Paris-Sud

Abstract:
On one hand, we will explain how a study of GL(2,Z)-orbits of square-tiled surfaces helps finding formulae for the number of pairs of permutations in Sn with a given commutator. On the other hand, we will show that a study of permutations which are commutators of generating pairs gives an asymptotic bound for the number of GL(2,Z)-orbits of square-tiled surfaces with a large monodromy group.

“Critical points" for surfaces diffeomorphisms, abundance of periodic orbits and structural stability

Enrique Pujals - IMPA

Abstract:
We will revisit a notion of critical set for surfaces diffeomorphisms and using them we will discuss some simple question related to the C □-stability conjecture and the problem of density of periodic orbits for C □-generic diffeos.

Remark: Discussions with S. Crovisier, A. Kocsard and A. Koropecki.

Perturbations of Roth type interval exchange maps and translation Surfaces

Jean-Christophe Yoccoz - Collège de France

Abstract:
We discuss a linearization theorem (obtained in collaboration with S. Marmi and P. Moussa) for perturbations of interval exchange maps or linear flows on translation surfaces under the appropriate diophantine condition.

Diffeomorphisms with contracting center

Jiagang Yang – Fluminense Federal University

Abstract:
The notion of contracting center means, roughly, that all Lyapunov exponents along the center direction are negative. It was introduced by Bonatti, Viana to ensure the existence and niteness of physical measures. Since then, it became clear that maps with contracting center have several distinctive features, that justify their study as a separate class of systems.

In this talk, I will briey recall the history of the study of diffeomorphism with mostly contracting center, and show the recent developments of this theory. As applications, we explain how to look for stationary measures of random diffeomorphism over circle, and prove the regularity of the Lyapounov exponents of general C1+ dominated SL(2;R) cocycle over hyperbolic base. In particular, we and a mechanism for the collapse of physical measures and the explosion of basins.

Remark: This talk contains several joint works with M. Viana.

References

[1] C. Bonatti and M. Viana. SRB measures for partially hyperbolic systems whose central direction is mostly contracting. Israel J. Math., 115:157-193, 2000.
[2] M. Viana, J. Yang, Physical measures and absolute continuity for one-dimensional center direction, www.preprint.impa.br.
[3] M. Viana, J. Yang, Difeomorphisms with contracting center, preprint. Departamento de Geometria, Instituto de Matemática, Universidade Federal Fluminense, Niterói, Brazil E-mail address: yangjg@impa.br

Stable projections of cartesian products of regular Cantor sets.

Jorge Erick López Velázquez - IMPA

Abstract:
We prove that typical projections on lR^k of cartesian products of regular Cantor sets with sum of Hausdorff dimensions greater than **k** contains persistently open sets. Our result is a generalization of the Moreira-Yoccoz's theorem (2001) about stable intersections of two regular Cantor sets on the real line. This is a joint work with Carlos Gustavo Moreira.

Robust vanishing of all central Lyapunov exponents

Lorenzo J. Diaz - PUC-RJ

Abstract:
We describe C2-open sets of iterated function systems on arbitrary compact manifolds admitting fully supported ergodic measures all whose Lyapunov exponents vanish.
We also exploit the consequences for partially hyperbolic maps.

Remark: Joint work with J. Bochi and Ch. Bonatti.

Images of Julia sets that you can trust

Luiz Henrique de Figueiredo - IMPA

Abstract:
We present an algorithm for computing images of quadratic Julia sets that can be trusted in the sense that they contain numerical guarantees against sampling artifacts and rounding errors. We avoid point sampling by using interval arithmetic to classify entire squares in the complex plane. We avoid fixing an arbitrary limit for the number of iterations performed by using cell mapping and label propagation in graphs. This also avoids floating-point errors. As a result, our algorithm is able to robustly classify rectangles in the complex plane as being on either side of the Julia set. The union of the regions that cannot be so classified is guaranteed to contain the Julia set. Our algorithm computes a quad tree decomposition of the complex plane adapted to the Julia set which can be used for rendering and for approximating the area of the filled Julia set and the fractal dimension of the Julia set.

Remark: Joint work with Diego Nehab (IMPA), Jorge Stolfi (UNICAMP), and João Batista Oliveira (PUCRS)

Time 1 maps of geodesic flows

Marcelo Viana - IMPA

Abstract:
Time-1 maps of geodesic flows are known to be partially hyperbolic, stably ergodic diffeomorphisms. For nearby diffeomorphisms, the center foliation is seldom smooth: indeed, it is nonsingular if and only if the perturbation itself is the time-1 map of a smooth flow.

Remark: Joint with Artur Avila and Amie Wilkinson.

On the Hyperbolicity of Lorenz Renormalization

Marco Martens - SUNY, Stony Brook

Abstract:
Often, the study of a flow can be reduced to the study of discontinuous interval maps. The discontinuity is a major obstacle: the usual one-dimensional techniques do not apply easily. In particular, the theory for Lorenz maps is far from being complete. However, for infinitely renormalizable Lorenz maps of long-monotone-type the dynamics is well understood: there are no wandering intervals, they are ergodic, there attractors are Cantors sets with measure zero carrying a unique invariant measure.

These results rely on the study of hyperbolicity of the corresponding renormalization operator acting on the space of smooth Lorenz maps with arbitrary critical exponent $\alpha > 1$. This work is in collaboration with B. Winckler.

Some questions, problems and remarks regarding C dynamics

Martín Sambarino - Fac. de Ciencias del Uruguay

Abstract:
We will review some old and folklore problems in the C topolgy, specially relative to stability.

C^r dichotomies between Newhouse phenomena and dominated splittings, at homoclinic points.

Nicolas Gourmelon - Université Bordeaux 1

Abstract:
Newhouse showed that if a diffeomorphism **f** admits a homoclinic tangency, then there is a newhouse domain close to **f**, that is a C open set of diffeomorphisms containing **f** in its closure, where locally generically diffeomorphisms have infinitely many sinks or sources. Palis and Viana generalized that result to higher dimension, when the homoclinic tangency is sectionally dissipative.
This leads to the following general question by Gonchenko, Shilnikov and Turaev: when is it possible to perturb the dynamics in the neighborhood of a homoclinic point in order to reach a Newhouse domain? We give a complete answer: such C perturbation is possible if and only if the homoclinic point is not volume partially hyperbolic.

Rigidity of Critical Circle Map

Pablo Guarino - Facultad de Ingenieria y Agrimesura

Abstract:
We will discuss the rigidity problem for critical circle maps: in a joint work with Welington de Melo. We recently proved that two C^3 critical circle maps with the same irrational rotation number of bounded type and the same odd degree at the critical point are conjugate by a C^{1+ α} circle diffeomorphism, for some $\alpha > 0$ that only depends on the rotation number. The proof is based on the study of the dynamics of a suitable defined renormalization operator.

Zoology in the Hénon family from twin baby Hénon like attractors

Pierre Berger - CNRS-LAGA, Paris 13, IMPA

Abstract:
We construct a renormalization process inside any Hénon like family to have two Hénon like renormalized maps which attract almost every point in the complement of the basin of infinity. The parameter dependence of these two maps is mostly uncorrelated. As consequence, we have for every $n > 1$ the existence of a Hénon map which has exactly **n** attracting cycles. This gives also examples of Hénon like map with as unique attractors infinitely many sinks and an infinitely renormalizable Hénon like map.

Partial hyperbolicity and leaf conjugacy in nilmanifolds

Rafael Potrie – University of Uruguayan Republic

Abstract:
We study strong (pointwise) partially hyperbolic diffeomorphisms in 3-manifolds with nilpotent fundamental group. We show that if the diffeomorphism is transitive then it must be dynamically coherent and leaf conjugated to a linear model. This is joint work with Andy Hammerlindl.

Center foliation: Absolute continuity, Disintegration and Rigidity

J.R.Varão – University of São Paulo, São Carlos

Abstract:
We study the measure-theoretical properties of center foliations of volume preserving partially hyperbolic diffeomorphisms with one-dimensional center direction. Recent work of Avila, Viana, Wilkinson dealt with situations where the center leaves are compact or can be compactified in a suitable way. Using different techniques we focus on the non-compact case and obtain very different conclusions. For instance, in our context the disintegration of volume may be neither atomic nor Lebesgue. Such examples are found even among Anosov diffeomorphisms. Moreover, even an Anosov may have absolutely continuous center foliation without being C^1-conjugate to its linearization. We also discuss a rigidity result.

The Lorenz Equations: A Survey of Rigorous Results

Sheldon Newhouse - Michigan State University

Abstract:
The "Lorenz Equations" form a parametrized system of differential equations in R^3 which depend on three real parameters. This system is easy to simulate numerically, and one sees a wide variety of phenomenon. Even after some thirty years of investigation by many authors, there are relatively few mathematically rigorous results which have been obtained. In this lecture, we will present a survey of results both old and new (some of which require verified computational methods) concerning these equations. We outline some of the currently available methods and discuss several open problems. While our main focus is on the specific Lorenz system, many of the techniques discussed have applications to a wide variety of systems of differential equations.

Newhouse phenomenon and uniformity of extremal bundles

Sylvain Crovisier - Université Paris 13

Abstract:
In the early 70's Newhouse has shown that there exists a set of diffeomorphisms, generic inside a C^2 open set, which exhibit infinitely many sinks or sources. This phenomenon appears close to the set of diffeomorphisms having homoclinic tangencies. In a joint work with E. Pujals and M. Sambarino, we now provide a converse for the C^1-topology: any C^1-generic diffeomorphism which exhibits infinitely many sinks or sources can be approximated by a diffeomorphism with a homoclinic tangency.

The main step is to obtain a generalization in any dimension of the Mane-Pujals-Sambarino hyperbolicity theorem on the uniformity of one-dimensional extremal bundles for C^2 diffeomorphisms.

Measures with historic behavior

Vilton Pinheiro – Federal University of Bahia (UFBA)

Abstract:
We say that a point **x** (or its orbit $\mathcal{O}^{+}(x)$) has historic behavior if $$\lim_{n\to\infty}\frac{1}{n}\sum_{j=1}^{n-1}\varphi(f^{j}(x))$$ does not exist for some $\varphi\in C^{0}(\XX)$.

That is, \begin{center}$\frac{1}{n}\sum_{j=1}^{n-1}\delta_{f^{j}(x)}$ does not converges in the weak topology.\end{center}

This terminology was introduced by Ruelle in 2001. We will use Caratheodory measures to study points with historic behavior, relating those points to hyperbolicity or homoclinc tangencies.

On the geometry of horseshoes

Carlos Gustavo Moreira (IMPA) and Waliston Silva – Federal University of São João del Rei

Abstract
The recurrent compact criterion was introduced by Moreira and Yoccoz to prove that
Intersections of regular Cantor sets are dense in the region where the sum of their
Hausdor dimensions is bigger than 1". We adapt this concept to the context of horseshoes in dimension higher than 2 and prove that horseshoes with upper stable dimension bigger than 1 satisfy, typically and persistently, the adapted recurrent compact criterion. As con- sequences we present some geometric properties of these horseshoes. In particular, tipically and persistently, horseshoes with upper stable dimension higher than 1 present blenders.

Stationary spaces of discrete groups: an Abramov formula

Yuri Lima - Weizmann Institute of Science

Abstract:
Consider the action of a discrete group G on a compact space. While G does not necessarily preserve some probability measure, there always exists a stationary measure: one that is preserved "on the average". In this setting, the Furstenberg entropy of G quantifies how far the measure is from being invariant.

In this talk we will present a formula which relates this Furstenberg entropy with the one of induced actions of finite index subgroups of G. The concepts are intimately related to the random walk on G and the proof uses both dynamics and probability methods. As a corollary, we relate the random walk entropies of G and of its subgroups of finite index. This is a joint work with Yair Hartman and Omer Tamuz.

Fiber contracting maps versus Lorenz-like attractors

Maria José Pacífico – Federal University of Rio de Janeiro

Abstract:
We prove exponential decay of correlations for certain fiber contracting maps and deduce some applications on the statistical behavior of Lorenz-like flows.