Optimal Transport and Applications

Luigi Ambrosio

2019 Balzan Prize for Theory of Partial Differential Equations

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Luigi Ambrosio is Director of the Scuola Normale Superiore in Pisa, Italy.

Ambrosio's Balzan research project is inspired by his recent work at the interface of several areas, including calculus of variations, geometric measure theory, optimal transport and its applications to partial differential equations and to metric geometry. He considers this ability to bring techniques from one field to another an important ingredient of his research and will use the Balzan Prize to keep his research goals as broad and interdisciplinary as possible in a five-year project which combines traditional topics like optimal transport, partial differential equations and curvature bounds for metric measure spaces with new emerging topics like machine learning. Ambrosio's interests derive from the use of transport distances in the design of the so-called loss functions on the one hand, and from the many emerging connections between neural networks and partial differential equations (especially gradient flow type equations) on the other.

The prize proposed using the Balzan funds for two-year non-teaching postdoctoral research grants based on the experience developed at the Centro di ricerca matematica Ennio De Giorgi of the Scuola Normale Superiore in Pisa. The positions were advertised internationally, and the project also involves international travel, conferences, and publication of research results in leading mathematical journals. As planned the funds of the Balzan project have been used to support the research of young mathematicians working in areas related to Optimal Transport and Analysis in Metric Spaces. For two years, it funded the research of Enrico Pasqualetto (now holding a tenure-track position at Jzvaskyla University in Finland. PhD student Camillo Brena has been involved in the field of differential calculus on metric measure spaces, which Ambrosio opened. In November 2023, a one-year research grant was awarded to Dr. Sumiya Baasandor. All researchers have worked or are still working under Ambrosio's supervision. Part of the funds has also been used to support two local initiatives in Pisa in January 2022 (postponed from 2020) and to provide a small travel grant to Camillo Brena. Remaining funds will support young researchers who collaborate with Ambrosio or carry out their own research work under his supervision.

Published papers where the support of the project is acknowledged:

[1] G. Antonelli, E. Pasqualetto, M. Pozzetta, and D. Semola, *Asymptotic isoperimetry on non collapsed spaces with lower Ricci bounds.* **Math. Ann.**, (2023), https://doi.org/10.1007/s00208-023-02674-y, 54 pp.

[2] D. Luˇci´c and E. Pasqualetto, *The metric-valued Lebesgue differentiation theorem in measure spaces and its applications*. **Adv. Oper. Theory**, 8 (2023),

https://doi.org/10.1007/s43036-023-00258-w, 51 pp.

[3] F. Essebei and E. Pasqualetto, *Variational problems concerning sub-Finsler metrics in Carnot groups.* **ESAIM Control Optim. Calc. Var.**, 29 (2023), https://doi.org/10.1051/cocv/2023006, 31 pp.

[4] E. Pasqualetto, *A short proof of the existence of master test plans*. **Arch. Math.**, doi.org/10.1007/s00013-022-01796-0, (2022), 8 pp.

[5] D. Luˇci´c and E. Pasqualetto, *Gamma-convergence of Cheeger energies with respect to increasing distances*. **J. Math. Anal. Appl.**, 515 (2022), no. 1, pp. 126415.

[6] T. Moisala and E. Pasqualetto, *Direct limits of infinite-dimensional Carnot groups*. **Math. Scand.**, 128 (2022), no. 2, pp. 160-200.

[7] E. Bru´e, E. Pasqualetto, and D. Semola, *Constancy of the dimension in codimension one and locality of the unit normal on RCD(K, N) spaces*. **Ann. Sc. Norm. Super. Pisa Cl. Sci.**, doi:10.2422/2036-2145.202110 007, (2022), 33 pp.

[8] F. Nobili, E. Pasqualetto, and T. Schultz, *On master test plans for the space of BV functions.* **Adv. Calc. Var.**, doi:10.1515/acv-2021-0078, (2022), 32 pp.

[9] D. Luˇci´c, E. Pasqualetto, and T. Rajala, Non-*Hilbertian tangents to Hilbertian spaces*. **Proc. R. Soc. Edinb. A: Math.**, 153 (2023), pp. 811-832.

[10] G. Antonelli, E. Pasqualetto, and M. Pozzetta, *Isoperimetric sets in spaces with lower bounds on the Ricci curvature.* **Nonlinear Anal. Theory Methods Appl.**, 220 (2022), pp. 112839.

[11] P. Bonicatto, G. Del Nin, and E. Pasqualetto, *Decomposition of integral metric currents*. **J. Funct. Anal.**, 282 (2022), no. 7, pp. 109378.

[12] T. Ikonen, E. Pasqualetto, and E. Soultanis, *Abstract and concrete tangent modules on Lipschitz differentiability spaces*. **Proc. Am. Math. Soc.**, 150 (2022), no. 01, pp. 327-343.

Papers accepted for publication where the support of the project is acknowledged:

[13] G. Antonelli, C. Brena, and E. Pasqualetto, *The Rank-One Theorem on RCD spaces*. **Anal. PDE** (in press), 41 pp.

Preprint where the support of the project will be acknowledged:

[14] L. Ambrosio, C. Brena, and S. Conti, *Functions with bounded Hessian- Schatten variation: density, variational and extremality properties*. Preprint arXiv:2302.12554 (2023), 49 pp.

[15] G. Antonelli, C. Brena, and E. Pasqualetto, Subgraphs of BV functions on RCD spaces. Preprint arXiv:2209.00645 (2022), 20 pp.

[16] N. Gigli, D. Luˇci´c, and E. Pasqualetto, *Duals and pullbacks of normed modules*. Preprint arXiv:2207.04972 (2022), 35 pp.

[17] M. Luˇci´c, E. Pasqualetto, and I. Vojnovi´c, *On the reflexivity properties of Banach bundles and Banach modules*. Preprint arXiv:2205.11608 (2022), 24 pp.

[18] G. Antonelli, E. Pasqualetto, M. Pozzetta, and D. Semola, *Sharp isoperi- metric comparison on non-collapsed spaces with lower Ricci bounds*. Preprint arXiv:2201.04916 (2022), 47 pp.

[19] T. Ikonen, D. Luˇci´c, and E. Pasqualetto, *Pullback of a quasiconformal map between arbitrary metric measure spaces*. Preprint arXiv:2112.07795 (2021), 22 pp.

[20] S. Di Marino, D. Luˇci´c, and E. Pasqualetto, *Representation theorems for normed modules*. Preprint arXiv:2109.03509 (2021), 46 pp.

[21] E. Caputo, N. Gigli, and E. Pasqualetto, *Parallel transport on non-collapsed RCD* (K, N) *spaces*. Preprint arXiv:2108.07531 (2021), 56 pp.

[22] E. Pasqualetto and T. Schultz, *Ultralimits of pointed metric measure spaces*. Preprint arXiv:2102.11365 (2021), 65 pp.