Computing Three Dimensional Fluids

Dennis Parnell Sullivan

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In order to compute fluid motion, any continuum fluid model must be discretized in terms of finitely many parameters. Discretizing space by dividing it into cells was Poincaré's starting point when he invented topology to study qualitative dynamical systems just over one hundred years ago. In the middle of the twentieth century, great advances were made in algebraic topology, which is also based on these cells. These advances are related to the algebraic products that are involved in the discretization process for the nonlinear term of the fluid models.

When discretizing continuum models, certain algebraic symmetry in the continuum models is broken. This loss of symmetry is repaired by a hierarchy of corrections based on algebraic topology. These corrections are similar to the Feynman diagrams used in the algorithms to compute physical effects in quantum theories.

Sullivan and his student colleagues have been engaged in understanding these corrections and building theoretical machinery for fluid computations based on these ideas. This work led to the revelation/understanding that different ways of discretizing various rewritings of the continuum model which are equivalent at the ideal level can be inequivalent at the discrete level.

There is coherence, however, if one allows for the extended sequence of corrections alluded to above. Systematically testing fluid data against the various algorithms in terms of these extended corrections would be beneficial. With the second part of his Balzan Prize, Sullivan has initiated testing the practical aspects of this theoretical work.

The construction of the algorithm for computing incompressible fluid motion, which replaces the continuum language by that of combinatorial topology, has been completed. One interesting point is that the algorithm is first derived from a tautologous conservation principle on an FCC lattice which is multiply covered by all cells of edge length twice the lattice scale. The algorithm is, however, NOT derived from the continuum model but derived directly. However, the primary or basic algorithm tends as the scale tends to zero to the continuum model written in the Leray form. This is the form which allows the definition of generalized and statistical solutions. The idea now is to imagine the algorithm to be written at such a small scale [above the atomic scale] that the momentum vectors on each face are essentially constant. Then the lattice and algorithm are amalgamated as in Wilson renormalization to reach a coarser level where computation is feasible. The cumulants of this process can be fit with the above mentioned hierarchical corrections [Thesis of Nissim Ranade]. A description of the basic or starting algorithm will appear in the memorial volume for Jean-Christophe Yoccoz, Collège de France, entitled “Lattice Hydrodynamics”. See also Dennis Sullivan’s presentation of “Lattice Hydrodynamics” at the Simons Center for Geometry and Physics Video Portal.

Sullivan’s Balzan research project is primarily based at Stony Brook University, with parts being carried out at the Graduate Center of the City University of New York. The cooperation with Theodore Drivas of Princeton University marks a new beginning for the project. A conference, “Real Fluids in dim ≤ 3 | Complex Manifolds in dim ≥ 3”, was held at the CUNY Graduate Center in April 2018. For further details, see simons-mathfest2018.ws.gc.cuny.edu.